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BOOK of ABSTRACTS

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Strongly Perturbing the Rössler Attractor: a case for stochastic-like resonance and its biological relevance

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The effect of noise in non-linear dynamical systems, although extensively studied, never ceases to surprise us with its unexpected rich repertoire of counter intuitive results. The delay of bifurcations, its stabilizing effects and phenomena such as stochastic resonance add to the enrichment of dynamical behaviour of non-linear systems when perturbed by randomness. The case we present here consist of a strong parametric perturbation of the Rössler system in its chaotic regime since this system serves as an archetypal example of chaotic dynamics after the seminal pioneering work of L. Shilnikov. We report the detection a stochastic resonance-like phenomenon and discuss the challenges of its mathematical description. We shall also relate this kind of parametric perturbations' utility for biologically important phenomena like stochastic circuit switching in genes and neural systems and touch on the subject of decision mechanisms utilizing a stochastic switch. Finally we will briefly explore its relevance to symbolic dynamics generated by coarse grained dynamical systems.

On trajectory attractor approximations of the 3D Navier-Stokes system by various hydrodynamical alpha-models

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An α-model is a mollification of the 3D Navier–Stokes (NS) system in which the smoothing is performed by some predefined filtering of the velocity arguments in the nonlinear term of the original NS system [1, 2]. Examples of such systems are: the Lagrangian averaged NS-α model or viscous Camassa–Holm equations, the Leray-α model, the simplified Bardina-α model. It was demonstrated analytically and computationally in many works that these α-models are useful tools in the study of the motion of large eddy currents. It was also proved that the Cauchy problems for the mentioned above α-models are well-possed and they possess global attractors [3, 4].

In the present work, we study the limits as $\alpha \to 0+$ for the long-time dynamics of various α-models of viscose incompressible fluid and their relations with the trajectory attractor of the exact 3D NS system. An α-models is characterized by its nonlinear term that approximate and regularize in some sense the standard bilinear term of the classical 3D NS system. We partition the considered α-models into two classes depending on the orthogonal properties of their mollifying nonlinear terms. We show that attractors of α-models from Class I attracts the trajectories stronger than the attractors of α-models from Class II.

We consider bounded (in the energy norm) families of solutions of a given α-model for $0 < \alpha \leq 1$. For $\alpha = 0$, we formally have the classical 3D NS system for which the uniqueness theorem (on the entire time semi-axis) of the (existing) weak solution of the Cauchy problem is not proved yet. However, for the 3D NS system, we can construct the trajectory attractor $\mathfrak{A}_0$ which describes the dynamics of the system in the corresponding local weak topology [5, 6].

For both classes of α-models, we prove that the bounded families of trajectories of the considered α-model converge to the trajectory attractor $\mathfrak{A}_0$ of the exact 3D NS system as time t tends to infinity and $\alpha \to 0+$ in the local weak topology.

In particular, we show that the trajectory attractor $\mathfrak{A}_\alpha$ of a given α-model converges to the trajectory attractor $\mathfrak{A}_0$ of the 3D NS system as $\alpha \to 0+$ in the considered local weak topology.

For all α-models, we have constructed the minimal limits $\mathfrak{A}_{\min} \subseteq \mathfrak{A}_0$ of their trajectory attractors $\mathfrak{A}_\alpha$ as $\alpha \to 0+$. We have proved that each set $\mathfrak{A}_{\min}$ is a compact connected component of the trajectory attractor $\mathfrak{A}_0$. Moreover, all sets $\mathfrak{A}_{\min}$ are strictly invariant with respect to time translation semigroup.

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Stability analysis of abstract systems of Timoshenko type

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We consider an abstract system of Timoshenko type

$$\rho_1\ddot{\varphi} + aA^{\frac{1}{2}}(A^{\frac{1}{2}}\varphi + \psi) = 0$$

$$\rho_2\ddot{\psi} + bA\psi + a(A^{\frac{1}{2}}\varphi + \psi) - \delta A^{\gamma}\theta = 0$$

$$\rho_3\dot{\theta} + cA\theta + \delta A^{\gamma}\dot{\psi} = 0$$

where the operator A is strictly positive selfadjoint. For any fixed real γ the stability properties of the related solution semigroup $S(t)$ are discussed. In particular, a general technique is introduced in order to prove the lack of exponential decay of $S(t)$ when the spectrum of the leading operator A is not made by eigenvalues only, which is always the case if its inverse A^{-1} is not compact.

Asymptotic behavior of dynamical systems arising in fluid mechanics

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We consider a system of equations modelling the evolution of an energetically isolated fluid system driven by external volume forces of various types. The existence of absence of attractors for such a system is discussed. We also show stabilization to equilibrium enforced by highly oscillating external forces with growing amplitude.

Asymptotic properties of invariant measures for stochastically forced Boussinesq equations

Földes J. (USA)

Parabolic Equation of normal type connected with 3D Helmholtz system.

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We consider normal parabolic equations (NPE) connected with 3D Helmholtz equations whose nonlinear term B(v) is orthogonal projection of nonlinear term for Helmholtz system on the ray generated by vector v. We will describe the structure of dynamical flow corresponding to this NPE and explain why this NPE can be interesting. Our main goal is to study nonlocal stabilization problem for NPE introduced above by starting control supported on arbitrary fixed subset with nonempty interior. The main steps of solution to this problem will be discussed.

Approximation of groups, characterizations of sofic groups, and equations over groups.

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Sofic groups was defined in relation with the Gottscholk surjunctivity conjecture. Hyperlinear groups was introduced in relation with Connes' embeding conjecture. It is known that sofic groups are hyperlinear, the other inclusion is an open question.

Some famous conjecture in group theory (Kervaire, Gottscholk, Connes' embedding conjectures) are proved to hold for sofic groups. It is also known that some important classes of groups are sofic, for example, amenable, residually amenable, extensions of amenable by sofic, etc.

An open question is if all groups are Sofic (Hyperlinear).

Classically, sofic (hyperlinear) groups are defined as metric approximation by symmetric groups (unitary groups). It is possible to define metric approximation by different classes of groups.

By definition, the metric approximation depends on invariant length functions and a class of groups. The structure of the set of invariant length functions on a group depends on the algebra of the conjugacy classes of this group. The aim of the present talk is to define and investigate the notion of approximation based on products of conjugacy classes without direct use of any length functions. Such approximations will be called $\mathcal{K}$-approximations. Let Sym, Alt, Nil, Sol, Fin be the classes of finite symmetric, finite alternating, finite nilpotent, finite solvable and all finite groups, respectively. We show that the classes of Alt-approximable groups, Sym-approximable groups, and sofic groups coincide. Fin-approximable groups are called weakly sofic.

Dynamical properties of logistic equation with state-dependent delay

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Local dynamics of classical Hutchinson's equation

$$\dot{N} = \lambda N(1 - N(t-1)), \quad \lambda > 0$$

is well known. In this report we consider more general form of this equation, namely:

$$\dot{N} = \lambda N(1 - N(t - T(N))), \quad \lambda > 0, \qquad (1)$$

where function $T(N)$ plays the role of state-dependent delay, and discuss its local dynamical properties. Namely, we investigate dynamics of equation (1) in a small neighborhood of its positive equilibrium at λ close to critical value $\pi/2$.

We make the next assumptions on $T(N)$: it is analytical near $N = 1$, positive in its definition region, bounded by positive constant T_1 and $T(1) = 1$. The expansion of $T(N)$ in the Taylor formula is

$$T(N) = 1 - \alpha(N-1) - \beta(N-1)^2 + o\left((N-1)^2\right),$$

where α and β are nonzero parameters. All the considered solutions of the equation (1) are assumed to be bounded.

Using known local method we construct normal form for equation (1):

$$rz' = \mu z + \nu z|z|^2.$$

Then we analyze this obtained equation and determine values of parameters α and β in which supercritical Andronov – Hopf bifurcation occurs in equation (1(near positive equilibrium at λ close to critical.

Quasi-Feynman formulas for the one-dimensional Schrödinger equation with a bounded smooth potential via the Remizov theorem

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Quasi-Feynman formula is a representation of a function in a form which includes multiple integrals of an infinitely increasing multiplicity, see [1]. The first toy-model for the Remizov theorem (theorem 3.1 in [1]) was suggested by A.S.Plyashechnik. The model was a one-dimensional Schrödinger equation with a bounded smooth potential. We prove that the conditions of the Remizov theorem are satisfied and show the arising quasi-Feynman formulas.

Consider the Cauchy problem in $L^2(\mathbb{R})$

$$\begin{cases} \frac{i}{a}\frac{d\psi(t,x)}{dt} = -\frac{1}{2}\frac{d^2\psi(t,x)}{dx^2} + V(x)\psi(x); \quad t\in\mathbb{R}, x\in\mathbb{R} \\ \psi(0,x) = \psi_0(x); \quad x\in\mathbb{R} \end{cases}$$

Above a is a non-zero number, $0 \neq a \in \mathbb{R}$, and V a bounded function with bounded continuous derivative, $V \in C_b^1(\mathbb{R},\mathbb{R})$.

We show that the solution of this Cauchy problem can be obtained in the form of the quasi-Feynman formula

$$\psi(t,x) = \lim_{n\to\infty}\lim_{k\to\infty}\sum_{m=0}^{k}\sum_{q=0}^{m}\frac{(-1)^{m-q}i^m a^m n^m(\mathrm{sign}(t))^m}{q!(m-q)!}\left(\frac{n}{2\pi|t|}\right)^{q/2}\times$$

$$\times\underbrace{\int_{\mathbb{R}}\dots\int_{\mathbb{R}}}_{q}\exp\left\{-\frac{|t|}{n}\left[\frac{1}{2}V(x)+\sum_{p=1}^{q}V\left(x+\sum_{j=p}^{q}y_j\right)\right]-\frac{1}{2t}\sum_{j=1}^{q}y_j^2\right\}\times$$

$$\times f\left(x+\sum_{j=1}^{q}y_j\right)\prod_{p=1}^{q}dy_p.$$

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Bifurcation research and stabilization chaotic systems of Lorenz type

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The models described multiparameter systems of three differential equations of Lorentz type (model gyrostat and economic model of the average firm):

$$\dot{x} = -\sigma x + \delta y, \dot{y} = \mu x + \nu y - \beta xz, \dot{z} = -\gamma z + \alpha xy. \tag{2}$$

As a bifurcation parameters considered μ, ν, γ and the parameters α, β, δ, σ are fixed. For special points system built partition Space bifurcation parameters on the field according to the type of rough singular point of the linearized system. When crossing the border field of saddle-focus with positive real part couples complex conjugate roots going Andronov-Hopf bifurcation the birth of a stable limit cycle, followed by a cascade period-doubling bifurcations cycle and subharmonic cascade Sharkovskii ending cycle period of the birth of three. A further change in the parameters appear in the system cycles homoclinic bifurcation cascade leading to the formation strange attractor. Using systems and transformations evidence of calculations show the existence of homoclinic the trajectory of a saddle-focus, the destruction of which is the main homoclinic bifurcation cascade, and identify areas parameters in which it exists. Bifurcation diagrams, graphs Lyapunov exponents,saddle of graphics, fractal dimension of the strange attractor. Objectives stabilization of unstable singular points of these systems solved extended by the control system. The parameters control systems to ensure the stabilization of the singular point in the range of the main bifurcation parameter, covers an area of chaos.

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Attractors for the 2D damped Navier-Stokes system on large periodic domains and in R^2

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We consider the damped and driven Navier–Stokes system

$$\partial_t u + (u, \nabla)u + \nabla p + \alpha u = \nu \Delta u + g, \quad div\ u = 0. \tag{3}$$

with additional dissipative term αu, $\alpha > 0$, modelling the Ekman friction.

In the case of a periodic domain $x \in [0, 2\pi L]^2$ it was shown in [1] that the system possesses a global attractor $\mathcal{A}$ (in L^2) with finite fractal dimension

$$\dim_f \mathcal{A} \le \min\left(\sqrt{6}\,\frac{\|\operatorname{curl} g\| L}{\nu\alpha}\,,\ \frac{3}{8}\,\frac{\|\operatorname{curl} g\|^2}{\nu\alpha^3}\right). \tag{4}$$

We observe that both estimates are of the order $1/\nu$ as $\nu \to 0^+$ and this rate of growth of the dimension is sharp [1].

For the system (3) on the elongated periodic domain $x \in [0, L] \times [0, L/\gamma]$, $\gamma \ll 1$ we have the estimates (provided that $\alpha \ge (5/8)\nu/L^2$)

$$\dim_f \mathcal{A} \le \min\left(12\,\frac{\|\operatorname{curl} g\| L}{\sqrt{\gamma}\,\nu\alpha}\,,\ 6\left(\frac{1}{\pi} + \sqrt{\frac{2}{\pi}}\right)\frac{\|\operatorname{curl} g\|^2}{\nu\alpha^3}\right), \tag{5}$$

in which the rate of growth $1/\nu$ is also sharp as $\nu \to 0^+$ and $\gamma \to 0^+$ [2].

While the first estimates in (4), (5) blow up as $L \to \infty$, the second estimates survive. Therefore, one might expect that these estimates hold for $L = \infty$, that is, for $x \in R^2$, and a motivation of the present work [3] is to show that this is indeed the case.

Theorem. *Let $x \in R^2$ and let the right-hand side g belong to the scale of homogeneous Sobolev spaces $\dot{H}^s$, $s \in [-1, 1]$. Then*

$$\dim_f \mathcal{A} \le \frac{1 - s^2}{64\sqrt{3}}\left(\frac{1 + |s|}{1 - |s|}\right)^{|s|} \frac{1}{\alpha^{2+s}\,\nu^{2-s}}\|g\|^2_{\dot{H}^s}, \quad s \in [-1, 1].$$

In particular, for $s = 1$ we obtain

$$\dim_f \mathcal{A} \le \frac{1}{16\sqrt{3}}\frac{\|\operatorname{curl} g\|^2}{\nu\,\alpha^3}.$$

The last estimate up to a constant coincides with the second estimates in (4), (5) proving thereby our expectation.

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Normalization of equations with two delays of different order

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Consider the equation with two delays

$$\dot{x} + x = ax(t - T_1) + bx(t - T_2) + f(x.x(t - T_1), x(t - T_2)), \quad T_1 > T_2 > 0,$$

where $f(x, y, z)$ is nonlinear function ($f(0, 0, 0) = 0$). Main assumption is that both T_1 and T_2 are asymptotically large and $T_1 T_2^{-1}$ is large too. Let $T_1 = \varepsilon^{-1}$, where $0 < \varepsilon \ll 1$. Then $T_2 = \varepsilon^{-1}(k_0 + \varepsilon^\alpha k_1)$ ($\alpha > 0$). The problem to research is to determine the behavior of solutions in some small (but independed of ε) neighbourhood of zero equilibrium state. The method of investigations is so-called method of quasinormal forms.

We proof that if $|a| + |b| < 1$ then $z = 0$ is stable and if $|a| + |b| > 1$ then zero is unstable. So $|a| + |b| = 1$ is critical case.

In critical case we construct special evolutionary equations (quasinormal forms). Their non-local dynamics determines the local behavior of solutions of the original equations. The particular kind of quasinormal forms is highly depends on parameter α. There are three different situations: (1) $\alpha < 1$, (2) $\alpha = 1$ and (3) $\alpha > 1$. Also, there are important situation when b is small, so we have small multiplier at the term with largest delay.

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Periodic and chaotic dynamics of weakly nonlinear shock waves

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Weakly nonlinear multi-dimensional shock waves are characterized by small amplitude and weak curvature of the shock front. When such waves propagate in a chemically reacting gas, the energy released in chemical reactions can make them self-sustained (they are called detonations). We derive an asymptotic model for the dynamics of these waves from the compressible reactive Navier-Stokes equations. The resultant model in 2D and in dimensionless form is given by (Faria, L. M. and Kasimov, A. R. and Rosales, R. R., An asymptotic theory of weakly non-linear detonations, *http://arxiv.org/abs/1407.8466*, 2014)

$$\begin{aligned} u_t + uu_x + v_y &= -\frac{1}{2}T_x + \mu u_{xx} \\ v_x &= u_y \\ \lambda_x &= -k(1-\lambda)e^{\theta T} - d\lambda_{xx} \\ \kappa T_x + T &= u + q\lambda + qd\lambda_x. \end{aligned}$$

where u, v is the velocity field, T is the temperature, and $\lambda \in [0, 1]$ is the variable measuring the fraction of the chemical energy, q, released in the reactions. The parameters μ, κ, and d are coefficients of viscosity, heat conduction, and diffusion, respectively. Parameters k and θ characterize the heat release rate. This system is a generalization of the models of small disturbance unsteady transonic flow, weakly nonlinear acoustics (Zabolotskaya-Khokhlov (ZK) equation), and water waves (dispersionless Kadomtsev-Petviashvili (KP) equation). Without chemical and dissipative terms ($\mu = \kappa = d = q = 0$), our model reduces to $(u_\tau + uu_x)_x + u_{yy} = 0$, which is the same as ZK or dispersionless KP equation. The model predicts regular and irregular multidimensional patterns, and in 1D exhibits transition from steady and stable traveling waves to oscillatory traveling waves through a Hopf bifurcation as θ is increased. A cascade of period-doubling bifurcations leading to chaos is also observed.

On linear stability and dispersion for crystals in the Schrödinger-Poisson model

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We consider the Schrödinger-Poisson-Newton equations as a model of crystals. Our main results are the well posedness and dispersion decay for the linearized dynamics at the ground state. This linearization is a Hamilton system with nonselfadjoint (and even nonsymmetric) generator. We diagonalize this Hamilton generator using our theory of spectral resolution of the Hamilton operators with positive definite energy [**?**, **?**], which is a special version of the M. Krein-H. Langer theory of selfadjoint operators in Hilbert spaces with indefinite metric. Using this spectral resolution, we establish the well posedness and the dispersion decay of the linearized dynamics with positive energy.

Our key technical result is the energy positivity for the linearized dynamics with small elementary charge $e > 0$ under a novel Wiener-type condition on the ions positions and their charge densitities. We give examples of crystals satisfying this condition.

The main difficulty in the proof of the positivity is due to the fact that for $e = 0$ the minimal spectral point $E_0 = 0$ is an eigenvalue of infinite multiplicity for the energy operator. To prove the positivity we study the asymptotics of the ground state as $e \to 0$ and show that the zero eigenvalue $E_0 = 0$ bifurcates into $E_e \sim e^2$.

On asymptotic stability of kinks for relativistic Ginzburg-Landau equation

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We consider nonlinear relativistic wave equation in one space dimension

$$\ddot{\psi}(x,t) = \psi''(x,t) + F(\psi(x,t)), \quad x \in \mathbb{R}, \quad F(\psi) = -U'(\psi), \tag{6}$$

where $U(\psi)$ is a potential of Ginzburg-Landau type

$$U(\psi) \sim (\psi^2 - 1)^2/4.$$

The kink is a nonconstant finite energy solution of stationary equation

$$s(x) \sim \tanh x/\sqrt{2}.$$

The corresponding moving kinks or solitary waves

$$s_{q,v}(t) = s(x - vt - q), \quad q, v \in \mathbb{R}, \quad |v| < 1, \quad \gamma = 1/\sqrt{1 - v^2}$$

are the solutions to equation (1). Our main results are the following asymptotics

$$(\psi(x,t), \dot{\psi}(x,t)) \sim (s_{q_\pm, v_\pm}(x - v_\pm t - q_\pm), \dot{s}_{q_\pm, v_\pm}(x - v_\pm t - q_\pm)) + W_0(t)\Phi_\pm, \quad t \to \pm\infty$$

for solutions to (1) with initial states close to a solitary wave. Here $W_0(t)$ is the dynamical group of the free Klein-Gordon equation, $\Phi_\pm$ are the corresponding asymptotic states, and the remainder converges to zero in the "global energy norm" of the Sobolev space $H^1(\mathbb{R}) \oplus L^2(\mathbb{R})$.

The proof techniques depend on the spectral properties of the linearized equation and may be regarded as a modern extension of the Lyapunov stability theory. Crucial role in the proof play our results on dispersion decay for the corresponding linearized Klein-Gordon equations. We also construct an examples of nonlinear equations with prescribed spectral properties of the linearized dynamics.

Inertial manifolds for the 3D Cahn-Hilliard equation with periodic boundary conditions

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My talk will be devoted to the existence of an inertial manifold (IM) for the 3D Cahn-Hilliard equation with periodic boundary conditions. In general, the existence of an IMs requires strong spectral gap condition which is violated in our case. Nevertheless, it appears that corresponding IM can be constructed using the proper extension of the so-called spatial averaging principle introduced by G. Sell and J. Mallet-Paret. This is the joint work with Prof. Sergey Zelik.

From the Fermi-Pasta-Ulam model to the high-order nonlinear evolution equations

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Three coupled rotators: from Anosov dynamics to hyperbolic attractor

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The work presents an example of a system with chaotic dynamics built of three rotators by modifying a conservative system with hyperbolic Anosov dynamics [1]. Results of a computational study of chaotic dynamics are considered (portraits of attractors, time dependences of the variables, Lyapunov exponents, and spectra) and good correspondence is observed between the dynamics on the attractor of the proposed system with the reduced model, characterized by the Anosov dynamics at appropriately defined energy [2]. The work is supported in part by RFBR grant No 15-02-02893 and by RSF grant No 15-12-20035.

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The axially symmetric dynamics for dissipative parabolic equations on the sphere

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Dissipative scalar parabolic equations on an interval have well known dynamics, and in particular the attractors can be constructed explicitly. Such construction use the zero-dropping property and a permutation related to the equilibria equation.

We are interested in the dynamics of dissipative parabolic equations on the sphere. In particular, axially symmetric solutions can be reformulated as an equation on an interval. However, there is a coefficient that is singular at both boundary points. We adapt a method used by Chen and Poláčik to prove that the zero-dropping property still holds. Moreover, we show the difficulties of constructing a permutation as it was done by Fusco and Rocha. Lastly, it is shown how both this ingredients can be combined to construct the attractor explicitly, as it was done by Brunovský, Fiedler, Rocha and others.

Some recent results on tempered pullback attractors for non-autonomous variants of Navier-Stokes equations

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During the last years, there have been several different approaches to non-autonomous dynamical systems for time-dependent problems and their long-time behaviour. In this talk I will focus on pullback attractors associated to some variants of Navier-Stokes (NS) equations with time-dependent terms. Issues to be analyzed in the exposition will include non-local (time) effects, tempered universes and tempered behaviour, regularity, and even well-posedness of some problems related to NS with and without delay, and others like Navier-Stokes-Voigt.

This talk is based partially in some works done in collaboration with J. García-Luengo (Universidad de Sevilla, Spain), G. Planas (IMECC, Universidade Estadual de Campinas, Brazil), J. Real (Universidad de Sevilla, Spain), and J. C. Robinson (University of Warwick, UK).

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Some generalizations of the Cahn-Hilliard equation

Alain Miranville (France)

Our aim in this talk is to discuss the qualitative behavior (existence of finite-dimensional attractors and blow up in finite time) of variants of the Cahn-Hilliard equation. Such equations arise in the context of image inpainting and biology.

Multistability in quasiperiodically driven Ikeda map

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It is well known that dynamical systems with weak dissipation can demonstrate a great number of attractors [1]. In this work mechanisms of phase space structure changes for the system with weak disipation while the quasiperiodical influence to the system is induced have been investigated using the Ikeda map [2].

The investigated map is given by

$$E_{n+1} = A(1 + \varepsilon \sin(\Omega \cdot \varphi \cdot n)) + BE_n \exp(i|E_n|^2 + i\varphi),$$

where A is control parameter, B is parameter of dissipation, ε is amplitude of external influence, Ω is frequency of influence, φ is phase.

In the work the structure of coexisting attractors and their evolution while ε is changed in the case of weak dissipation are investigated. It is shown that the number of coexisting attractors decreases in comparison with the case of absence of external influence. It occurs due to the finite size of torus attractor in contrast to periodical attractor. The attractor number dependence on the ε is studied for different values of A.

The evolution of attractor basins boundaries while quasiperiodical influence is appended has been studied.

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Invariant measures and attractors of non-autonomous Frenkel-Kontovora models

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Dissipatively driven Frenkel-Kontorova (FK) model is a system of infinitely many coupled particles in a periodic potential, with over-damped (or gradient) dynamics, important in e.g. physical applications. It is related to scalar reaction-diffusion equations, and can be understood as its discrete-space analogue. We develop ergodic theory for non-autonomous FK models, and in particular show that the union of supports of all space-time invariant measures is at most two-dimensional set, mainly consisting of synchronized orbits. This explains experimentally and numerically observed behavior. We then focus on the ratchet equations (i.e. oscillating site-potential, no external force), and give rigorous sufficient conditions for existence of transport, again explaining results of experiments.

Diffusion in Hilbert space equation solved by Feynman formula

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In this talk I present the results of [1], which are the continuation of [2] and are also available in [3].

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On a fractional Cahn-Hilliard equation

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In this talk we will present some results related to existence, regularity, and long-time behavior of solutions to a fractional version of the Cahn-Hilliard equation settled in a smooth bounded domain $\Omega \subset \mathbf{R}^3$. More precisely, we will consider the case where diffusion is ruled by the so-called "restricted Dirichlet fractional Laplacian", meaning that homogeneous Dirichlet conditions of "solid" type are assumed on the whole of $\mathbf{R}^3 \setminus \Omega$. In particular, we will show that, under suitable conditions, the ω-limit set of any solution trajectory consists of a single point. The proof of this fact relies on a new "fractional" version of the Simon-Łojasiewicz inequality. The results presented in this talk have been obtained in collaboration with Goro Akagi (University of Kobe) and Antonio Segatti (University of Pavia).

Controllability implies ergodicity

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In this talk, we discuss the interconnection between controllability properties of a dynamical system and large-time asymptotics of trajectories for the associated stochastic system. We begin with a result on the finite-dimensional case which applies to differential equations on a smooth Riemannian manifold. We show how the approximate controllability to a given point and solid controllability imply the uniqueness of a stationary measure and exponential mixing in the total variation distance. We next turn to problems in infinite dimension and formulate a sufficient condition (in terms of controllability properties) for the exponential mixing in the Kantorovich–Wasserstein distance. This result applies, for instance, to the 2D Navier–Stokes system driven by a random force acting on the boundary. Finally, we formulate some open problems on controllability properties of the Navier–Stokes system, which would have interesting applications in the ergodic theory of the associated random flow.

Intrinsic Shape of Non-Saddle Sets

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Asymptotically stable attractors are only a particular case of a large family of invariant compacta whose global topological structure is regular. We devote this talk to introducing this class of compacta, the non-saddle sets. Attractors and repellers are examples of non-saddle sets. The main aim of this presentation is to generalize the well known theorem for shape of global attractors to non-saddle sets using the intrinsic approach to shape.

Stochastic bifurcations in the Hindmarsh-Rose model

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We study the effects of random disturbances on the Hindmarsh-Rose model [1] of neuron activity:

$$\begin{aligned} \dot{x} &= y - x^3 + 3x^2 + I - z + \varepsilon \dot{w}, \\ \dot{y} &= 1 - 5x^2 - y \\ \dot{z} &= r(s(x - x_0) - z), \end{aligned} \tag{7}$$

where x is a membrane potential, variables y, z describe ionic currents, I is an external current; $0 < r \ll 1$ is a time scale parameter; s, x_0 are other parameters; w is a standard Wiener process with $\mathrm{E}(w(t) - w(s)) = 0$, $\mathrm{E}(w(t) - w(s))^2 = |t - s|$ and ε is a noise intensity.

We fix $r = 0.002$, $s = 4$, $x_0 = -1.6$ and examine the dynamics of the system under variation of the parameter I.

Due to the strong nonlinearity, even the original deterministic ($\varepsilon = 0$) system demonstrates very diverse complex dynamic regimes. Random perturbations considerably affect the properties of neuronal systems. Even small stochastic fluctuations can lead to a significant qualitative changes in the nonlinear dynamics of such systems.

We consider a parametrical zone, where the deterministic system demonstrates both mono- and bistable dynamic regimes. In the parametric region, where the only attractor of the deterministic system is stable equilibrium, the phenomenon of stochastic generation of high-amplitude oscillations is observed. In the parametric zone of the coexisting stable equilibrium and limit cycle, the system exhibits noise-induced transitions between the attractors.

These stochastic phenomena are confirmed by the changing of the probability density distribution of random trajectories. So, under the random disturbances, the system demonstrates P-bifurcations related to the qualitative change of the distribution of random states.

An exhaustive probabilistic description of the stochastic attractors is given by Kolmogorov-Fokker-Planck equation. However, the direct usage of it is very difficult even for the simplest cases. Various approximations and asymptotics can be used. For the analysis of the stochastic phenomena in the Hindmarsh-Rose model, we suggest an approach combining stochastic sensitivity function technique and confidence domains method [2].

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Dynamics of extended gradient systems

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Extended gradient systems are dynamical systems on unbounded domains, which when suitably restricted to a finite domain are gradient-like. We present general results on the structure of ω-limit sets, invariant measures, and stability of equilibria and invariant manifolds. We then apply the results to a series of examples, illustrating similarities and differences with gradient-like systems.

The general theory yields some new results in the following examples on unbounded domains: dynamics of various reaction-diffusion equations (with energy, entropy, and Zelenyak/Matano/Fiedler/Rocha-like Lyapunov functions); dynamics of unforced Navier-Stokes equation in 2D; construction of orbits and measures of Lagrangian systems and related formally gradient dynamics of the action functional; and finally asymptotics of some Markov chains on infinite lattices and related phase transitions. For such systems we also often obtain new bounds on relaxation times on bounded domains, independent of the domain size.

A larger part of the work is a joint work with Thierry Gallay.

Chaos, hyperchaos and quasiperiodicity in the system of coupled Toda oscillators

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Coupled oscillator systems play important role in the study of chemical, biological and physical processes [1]. Synchronization is a fundamental nonlinear phenomenon occurring via interaction of self-sustained oscillators. However, one can consider the dynamics of interacting dissipative oscillators with external driving force, which have stable point and have not stable limit cycle in phase space without driving. In the series papers [2-5] was revealed, that in ensembles of coupled dissipative oscillators can be observed such phenomena as synchronization, quasiperiodic oscillations and other.

In the present paper we consider such problem on the example of Toda oscillator. We consider coupled two and three oscillators excited by antiphase periodic harmonic signal. We discuss the features of occurrence quasiperiodic oscillations in such system and attempt to realize three-frequencies quasiperiodic oscillations. In the such systems was obtained chaotic oscillations with different amount of Lyapunov exponents (one, two and three). We consider different scenarios of transitions to different chaotic regimes. In order to get enough complete picture of dynamics regimes of such systems, were considered systems with different topology of coupling: chain and ring. For this systems chart of Lyapunov exponents on the different parameter plane were constructed.

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УПРАВЛЕНИЕ АТТРАКТОРА ПЛЫКИНА МЕТОДОМ ПИРАГАСА

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Как известно, хаотические системы чрезвычайно чувствительны к внешним воздействиям. Эта особенность послужила предпосылкой для создания новых методов управления нелинейными системами и подавления в них хаоса. В данной работе изучается возможность стабилизации хаотических колебаний в системах с гиперболическим типом аттрактора посредством обратной связи и синусоидального возмущения.

Множество Λ называется гиперболическим аттрактором динамической системы, если Λ — замкнутое топологически транзитивное гиперболическое множество и существует такая окрестность $U \supset \Lambda$, что $\Lambda = \cup_{t\geq 0} f^n U$. К хорошо известным относятся гиперболический аттрактор Плыкина. Гиперболический аттрактор Плыкина располагается на двухмерной области $T = S^2$, где S^2 — единичная окружность. Тогда $f : \mathbb{T} \mapsto \mathbb{T}, f(x, y, z) = (\cos\varphi \sin\phi, \sin\varphi \sin\phi, \cos\phi)$, где значение $k > 2$, и представляет собой подмножество $\mathbb{T} \subset \mathbb{R}^3$.

В настоящее время,к гиперболическим аттракторам типа Плыкина [1] проявлен большой интерес, при моделировании сердечной аритмии и атмосферных процессов. Аттрактор Плыкина представлен следующей системой уравнений:

$$\begin{cases} \dot{X} = -2\epsilon Y^2 \Omega_1 (\cos(\omega_2 \cos\omega_1 t) - X\sin(\omega_2\cos\omega_1 t)) + \\ \qquad kY\Omega_2(\cos(\omega_2\sin\omega_1 t) - X\sin(\omega_2\sin\omega_1 t))\sin\omega_1 t, \\ \dot{Y} = 2Y\Omega_1(X\cos(\omega_2\cos\omega_1 t) + 2^{-1}(1-X^2+Y^2)\sin(\omega_2\cos\omega_1 t)) - \\ \qquad k\Omega_2(X\cos(\omega_2\sin\omega_1 t) + 2^{-1}(1-X^2+Y^2)\sin(\omega_2\sin\omega_1 t))\sin\omega_1 t + D(K,\tau), \\ \Omega_1 = (2X\cos(\omega_2\cos\omega_1 t) + (1-X^2-Y^2)\sin(\omega_2\cos\omega_1 t))(1+X^2+Y^2)^{-2}, \\ \Omega_2 = (-2X\sin(\omega_2\sin\omega_1 t) + (1-X^2-Y^2)\cos(\omega_2\sin\omega_1 t))(1+X^2+Y^2)^{-1} + 2^{-1/2}. \end{cases}$$

В настоящей работе показано, что посредством обратной связи Y и временной задерки τ вида $D(K,\tau) \mapsto K(Y(t-\tau) - Y(t))$ можно выводить данную систему на регулярный, хаотический и циклический режим.

Данный метод Пирагаса может быть использован в управлении и для других типов хаотических динамиких моделий аттракторов [2].

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Анализ стохастической динамики в 2D-логистическом отображении

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Данная работа посвящена исследованию двумерного логистического отображения [1] в присутствии внешних случайных возмущений

$$\begin{aligned} x_{t+1} &= (1-\lambda)x_t + 4\lambda y_t(1-y_t) + \varepsilon\xi_t \\ y_{t+1} &= (1-\lambda)y_t + 4\lambda x_t(1-x_t) + \varepsilon\eta_t, \end{aligned} \tag{8}$$

где ξ_t, η_t – независимые гауссовские случайные величины с параметрами $\mathrm{E}\xi_t = 0$, $\mathrm{E}\eta_t = 0$, $\mathrm{E}\xi_t^2 = 1, \mathrm{E}\eta_t^2 = 1$, а величина ε характеризует интенсивность возмущений.

В детерминированной модели существует четыре равновесия, два из которых для $0 < \lambda < 0.4$ являются устойчивыми, два других - всегда неустойчивые (седла). При $\lambda = 0.4$ происходит бифуркация Неймарка-Сакера и рождаются две сосуществующие замкнутые инвариантные кривые. Для исследования динамики изменения фазовых портретов детерминированной модели в зависимости от параметра построена бифуркационная диаграмма. На бифуркацинной диаграмме можно отметить области с регулярной динамикой, включающей разнообразные аттракторы и зоны, содержащие хаотические режимы. В данной работе исследовались зоны равновесий, замкнутых инвариантных кривых и дискретных 7-циклов. Изменение степени устойчивости аттракторов иллюстрируют показатели Ляпунова. В зоне существования замкнутой инвариантной кривой исследованы число вращения и секторная плотность.

Под влиянием шума стохастическая траектория покидает детерминированный аттрактор и образует вокруг него облако случайных состояний. Анализ распределения случайных состояний опирается на теорию функции стохастической чувствительности [2]. Детально исследована стохастическая чувствительность аттракторов модели и конфигурация доверительных областей. Исследованы коэффициенты чувствительности аттракторов в зависимости от параметра. Параметрически исследованы индуцированные шумом переходы от порядка к хаосу.

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SEMILINEAR PARABOLIC EQUATIONS WITHOUT INERTIAL MANIFOLD

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Инерциальное многообразие (ИМ) полулинейного параболического уравнения (ППУ) это гладкая конечномерная инвариантная поверхность в фазовом пространстве, содержащая глобальный аттрактор и экспоненциально притягивающая все траектории при большом времени. Сужение уравнения на ИМ представляет собой ОДУ, описывающее финальную динамику системы. Установить существование ИМ удаётся для узкого класса ППУ, тогда как известные примеры его отсутствия выглядят искусственно и не связаны с задачами математической физики.

Абстрактное ППУ в вещественном сепарабельном бесконечномерном гильбертовом пространстве $(X, \|\cdot\|)$ имеет вид

$$\partial_t u = -Au + F(u) \tag{1}$$

с линейным положительно-определённым оператором A, компактным A^{-1}, и гладкой нелинейной функцией $F : H \to X$, где $H = D(A^\alpha)$, $0 \le \alpha < 1$, $\|u\|_H = \|A^\alpha u\|$. Считаем, что (1) порождает гладкий диссипативный полупоток в H. Примеры отсутствия ИМ у ППУ строятся [1–3] на следующей основе. Для стационарных точек $u \in E \subset H$ спектр $\sigma(T(u))$ оператора $T(u) = F'(u) - A$ в X состоит из конечнократных собственных значений λ и число (с кратностью) $l(u)$ положительных λ в $\sigma(T(u))$ конечно. Пусть $E_- = \{u \in E : \sigma(T(u)) \cap (-\infty, 0] = \phi\}$.

ЛЕММА. *Если аттрактор уравнения* (1) *с нелинейностью* $F \in C^1(H, X)$ *содержится в инвариантном конечномерном* C^1*-многообразии* $M \subset H$, *то для любых* $u_0, u_1 \in E_-$ *число* $l(u_0) - l(u_1)$ *чётно.*

Рассмотрим интегро-дифференциальное уравнение

$$u_t \;=\; ((I+B)\,u_x)_x \;+\; f(x,u,u_x), \tag{2}$$

на единичной окружности Γ с $X = L^2(\Gamma)$. Здесь $I = \mathrm{id}$, $x \in \Gamma$,

$$(Bh)(x) \;=\; \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \ln\left|\sin\frac{x+y}{2}\right| h(y)\,dy$$

для $h \in X$, определённая на $\Gamma \times R^2$ функция $f(x,s,p)$ – бесконечно гладкая, но не аналитическая. Оператор $I+B$ играет роль нелокального коэффициента диффузии. Положим $Au = u - u_{xx}$.

ТЕОРЕМА ([1]). *При подходящем выборе функции* f *уравнение* (2) *порождает диссипативный* C^1*-полупоток в* $H = D(A^\alpha)$, $\alpha \in (3/4, 1)$, *причём его аттрактор не содержится ни в каком инвариантном конечномерном* C^1*-многообразии* $M \subset H$.

Фактически, строится функция f такая, что уравнение (2) имеет стационарные решения $u_0, u_1 \in E_-$ с $l(u_0) = 0$ и $l(u_1) = 1$.

Для *уравнений реакции-диффузии* известны [2,4] примеры отсутствия ИМ с условиями гиперболичности. Пусть для $u \in E$ прямая $\mathrm{Re}\lambda = \gamma$ лежит в $\varrho(T(u))$ и $H(u,\gamma)$ – инвариантное подпространство оператора $T(u)$, отвечающее части $\sigma(T(u))$ с $\mathrm{Re}\lambda > \gamma$. Инерциальное многообразие размерности n нормально гиперболично (на E), если $\dim H(u,\gamma) = n$ $\forall u \in E$ и $\gamma = \gamma(u) < 0$. Пользуясь результатами [2], можно построить диссипативную систему УРД

$$\partial_t u_1 \;= \Delta u_1 + f_1(u_1,u_2), \quad \partial_t u_2 \;= \Delta u_2 + f_2(u_1,u_2)$$

в кубе I^3 с условием Неймана на границе и полиномиальной нелинейностью (f_1, f_2), не допускающую нормально гиперболического ИМ в $C(I^3; R^2)$. При этом, по сравнению с аналогичного типа классическим контрпримером [4] размерность задачи понижается с четырёх до трёх и нелинейная часть не зависит от $x \in I^3$.

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About Boltzmann's entropy, Sanov's entropy and their relations

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Assume that some macrosystem can stay at different states characterized by the vector $\vec{n}$ with nonegative integer components (filling numbers). Let us assume that in this system the following reactions may occur:

$$\vec{n} \to \vec{n} - \vec{\alpha} + \vec{\beta},\ (\vec{\alpha}, \vec{\beta}) \in J$$

Following Leontovich (1934), let us introduce intensity of the reaction:

$$\lambda_{(\vec{\alpha},\vec{\beta})}(\vec{n}) = \lambda_{(\vec{\alpha},\vec{\beta})}(\vec{n} \to \vec{n} - \vec{\alpha} + \vec{\beta}) = N^{1-\sum\limits_i \alpha_i} K_{\vec{\beta}}^{\vec{\alpha}}(\vec{n}/N) \prod_{i:\alpha_i>0} n_i \cdot \ldots \cdot (n_i - \alpha_i + 1),$$

where $K_{\vec{\beta}}^{\vec{\alpha}}(\vec{n}/N) \geq 0$ is a constant of reaction. Note that in applications it is always assumed that

$$\sum_i n_i(t) \equiv N \text{ (N is often called the scalling parameter).}$$

Thus $\lambda_{(\vec{\alpha},\vec{\beta})}(\vec{n})$ is a probability of the reaction $\vec{n} \to \vec{n} - \vec{\alpha} + \vec{\beta}$ to take place in the unit of time. On the macrolevel this corresponds to the law of mass action (Guldberg-Vaage (1864)).

In this work we assume that number of states $m = dim\ \vec{n}$, number $|J|$ and constants $K_{\vec{\beta}}^{\vec{\alpha}}(\vec{n}/N)$ of reactions may depend on N (in contrast to [1]). Even so we additionally assume that $m \ll N$ that is necessary to support application of the Stirling formula upon obtaining variational principle (maximum of entropy). Let us proceed to the theorem which bridges Boltzmann's entropy (Lyapunov's function of scaled kinetic dynamic) and Sanov's entropy (Sanov's type function in a high probability deviations inequality).

Theorem 1 *Let there exists such a function $H(\vec{c})$ that invariant (stationary) measure of described above Markov dynamic fulfills the following representation (in C^2):*

$$\mu(\vec{n}) = exp(-N \cdot (H(\vec{n}/N) + o(1))),\ N \to \infty.$$

Then $H(\vec{c})$ is Lyapunov's function for the following ODE system of Guldberg-Vaage:

$$\frac{dc_i}{dt} = \sum_{(\vec{\alpha},\vec{\beta})\in J} (\beta_i - \alpha_i) K_{\vec{\beta}}^{\vec{\alpha}}(\vec{c})\vec{c}^{\vec{\alpha}}, \ \vec{c}^{\vec{\alpha}} = \prod_j c_j^{\alpha_j}. \qquad (GV)$$

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Feynman and Quasi-Feynman formulae for higher order Schrödinger equation

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One parameter semigroup approximations [1] related to the higher order Schrödinger equation $\frac{\partial}{\partial t}\psi(t,x) = -a(-\triangle\psi)^N(t,x)$ with complex coefficient a are considered. For $N = 2$ such approximations were obtained in [2]. Similar results for $N > 2$ are presented in this talk.

Feynman formulae (i.e. considered semigroup is represented by limits of iterated integrals of elementary functions when multiplicity of integrals tends to infinity) obtained for $N > 2$ are shown in the first part of the talk. Feynman formulae are deduced for real positive coefficient a (heat-type equation). Different types of Feynman formulas are presented in this work: Lagrangin and Hamiltonian. Lagrangian Feynman formulae are suitable for computer modeling of the considered dynamics. Hamiltonian Feynman formulae are related to some phase space Feynman path integrals; such integrals are important objects in quantum physics. The main part of these formulae is proved with the help of the Chernoff theorem; some formulae are obtained on the base of the Iosida approximations. Feynman formulae definition was introduced by O.G. Smolyanov [3]. See also overviews [4-6].

The Remizov theorem [7-8] is used in the second part of the talk to prove Quasi-Feynman formulae for complex coefficients case in the equation. Quasi-Feynman formula is a representation of a function in a form which includes multiple integrals of an infinitely increasing multiplicity. The difference from a Feynman formula is that in a quasi-Feynman formula summation and other functions/operations may be used while in a Feynman formula only the limit of a multiple integral where the multiplicity tends to infinity is allowed. The definition of the Quasi-Feynman formula was presented by I.D. Remizov, and the words «Quasi-Feynman formula» was suggested by O.G. Smolyanov.

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Reduced ODE systems governing coarsening dynamics of dewetting liquid films

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In this talk an overview of certain classes of high-order degenerate parabolic PDEs describing dewetting process in thin liquid films and demonstrating long time coarsening of special localized metastable solutions is presented. As a part the reduction of the dynamics governed by thin film type equations onto an 'approximate' finite-dimensional invariant manifold is derived following the approach in [1]. This corresponds physically to the late phase evolution of thin liquid films dewetting on a solid substrate, where arrays of drops connected by an ultrathin film of thickness ϵ undergo a slow-time coarsening dynamics. Respectively, our asymptotic approximation of the corresponding invariant manifold in the limit $\epsilon \to 0$ is parametrized by a family of droplet pressures and positions.

Subsequently, reduced systems of ODEs for the dynamics on the manifold are derived for different slip regimes considered at the solid substrate. Subsequently, dependence of the coarsening rates (i.e. the law describing how fast the number of drops decreases in time) on the physical parameters is analyzed. In the limiting case of free suspended films existence of a threshold for the decay of initial distributions of droplet distances at infinity at which the coarsening rates switch from algebraic to exponential ones is shown.

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Fractals and path integrals in three-dimensional wave equation

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Let us consider three-dimensional wave equation for function $u(\vec{x},t)$:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \cdot \Delta u\,, \tag{1}$$

where a is phase velocity and Δ is Laplacian. This equation describes a lot of different physical phenomena.

It's easy to see that one can represent equation (1) as a system:

$$\frac{\partial \cdot_1}{\partial t} = -a \cdot \sqrt{-\Delta} \cdot \cdot_2 \qquad \frac{\partial \cdot_2}{\partial t} = a \cdot \sqrt{-\Delta} \cdot \cdot_1\,. \tag{2}$$

In this system $\cdot_1(\vec{x},t) \equiv u(\vec{x},t)$ and fractional operator $\sqrt{-\Delta}$ is Hermitian operator which may be called by analogy with quantum mechanics by 'absolute value of momentum operator'. This operator possesses by inverse operator $(-\Delta)^{-1/2}$ acting on arbitrary function $f(\vec{x})$ as follows:

$$(-\Delta)^{-1/2} f(\vec{x}) = \frac{1}{2 \cdot ®^2} \cdot \int \frac{f(\vec{x}\,') \cdot d^3 x'}{|\vec{x} - \vec{x}\,'|^2}\,. \tag{3}$$

It means in particular that function $\cdot_2(\vec{x},t)$ can be expressed from the first equation of system (2) via $\dfrac{\partial u(\vec{x},t)}{\partial t}$.

Using Pauli matrix

$$\pm_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{4}$$

one can rewrite system (2) as equation for two-dimensional vector $\cdot = (\cdot_1, \cdot_2)^T$:

$$i \cdot \frac{\partial \cdot}{\partial t} = a \cdot \sqrt{-\Delta} \cdot \pm_y \cdot \cdot\,. \tag{5}$$

Due to identity $\pm_y^2 = 1$ it's possible to find that this Schrödinger type equation has the next unitary operator of evolution:

$$\exp(-i \cdot t \cdot a \cdot \sqrt{-\Delta} \cdot \pm_y) = \frac{1-\pm_y}{2} \cdot \exp(i \cdot t \cdot a \cdot \sqrt{-\Delta}) + \frac{1+\pm_y}{2} \cdot \exp(-i \cdot t \cdot a \cdot \sqrt{-\Delta})\,, \tag{6}$$

therefore vector $\cdot(\vec{x},t)$ can be expressed via it's initial state $\cdot(\vec{x},0)$:

$$\cdot(\vec{x},t) = \int \Gamma(\vec{x},\vec{x}\,';t) \cdot \cdot(\vec{x}\,',0) \cdot d^3 x'\,, \tag{7}$$

where Green's matrix for equation (5) is equal to:

$$\Gamma(\vec{x},\vec{x}\,';t) = \frac{1-\pm_y}{2} \cdot G^*(\vec{x}\,',\vec{x};t) + \frac{1+\pm_y}{2} \cdot G(\vec{x},\vec{x}\,';t)\,. \tag{8}$$

It's easy to check that Green's function is:

$$G(\vec{x},\vec{x}\,';t) \equiv \langle \vec{x}\,|\exp(-i \cdot t \cdot a \cdot \sqrt{-\Delta})|\,\vec{x}\,' \rangle = \frac{i}{2 \cdot ®^2 \cdot a} \cdot \frac{\partial}{\partial t} \frac{1}{(\vec{x}-\vec{x}\,')^2 - (a \cdot t)^2 + i \cdot 0}\,. \tag{9}$$

On the other side Green's function (9) can be expressed by Feynman integral:

$$G(\vec{x},\vec{x}';t)=\int\limits_{\vec{Q}(0)=\vec{x}'}^{\vec{Q}(t)=\vec{x}} \exp[i\cdot\int\limits_0^t(\vec{P}(\tau)\cdot\dot{\vec{Q}}(\tau)-a\cdot|\vec{P}(\tau)|)\cdot d\tau]\cdot\prod_{\tau}\frac{d\vec{P}(\tau)\cdot d\vec{Q}(\tau)}{2\cdot\pi}. \tag{10}$$

In order to calculate path integral (10) one ought to divide interval of time $[0,t]$ on N equal parts and to approximate coordinates $\vec{Q}(\tau)$ and momenta $\vec{P}(\tau)$ by:

$$\vec{Q}(\tau)=\vec{Q}_j+(\vec{Q}_{j+1}-\vec{Q}_j)\cdot(\tau-\tau_j)/\Delta\tau,\quad \vec{P}(\tau)=\vec{P}_j,\quad \tau\in[\tau_j,\tau_{j+1}], \tag{11}$$

where $\tau_j = j\cdot\Delta\tau$, $\Delta\tau = t/N$, $j=\overline{0,N}$. In means in particular that coordinates may walk on fractal trees in R^3. Furthermore dynamics of momenta proves to obey to succession map $\vec{P}_{j+1}=\vec{F}(\vec{P}_j)$. If map $\vec{F}:R^3\to R^3$ satisfies to conditions of Williams-Hatchinson theorem [1] then momenta also form fractal set in R^3. On the other hand in such map also may take place chaotic behaviour for instance for generalized Henon map [2].

Thus representation of Green's function (9) by path integral (10) gives us the possibility to introduce quantum quasiparticle related with input equation (1) as object moving along fractal trajectories in six-dimensional phase space $(\vec{P},\vec{Q})$. In honour of outstanding physicist of the 20th century Richard Feynman we call this quasiparticle by 'feynmanon'. But we underline that initially wave equation (1) is purely classical. And appearance of quantum quasiparticle in our consideration is direct consequence of nonlocality in system (2).

In conclusion it should be noted that one can quantize massless scalar field $u(\vec{x},t)$ with the help of annihilation $\hat{c}(\vec{p})$ and creation $\hat{c}^+(\vec{p})$ operators [3]:

$$\hat{u}(\vec{x},t)=\int[\hat{c}(\vec{p})\cdot\exp(i\cdot\vec{p}\cdot\vec{x}-i\cdot a\cdot|\vec{p}|\cdot t)+h.c.]\cdot\frac{d^3p}{\sqrt{2\cdot a\cdot|\vec{p}|\cdot(2\cdot\pi)^{3/2}}}\ . \tag{12}$$

where measure corresponds to following Bose canonical commutative relations [3]:

$$[\hat{c}(\vec{p}),\hat{c}^+(\vec{p}')]=\delta(\vec{p}-\vec{p}'),\quad [\hat{c}(\vec{p}),\hat{c}(\vec{p}')]=0, \tag{13}$$

one can calculate the next operator:

$$\hat{H}=\int:\left[\frac{1}{2}\cdot\left(\frac{\partial\hat{u}(\vec{x},t)}{\partial t}\right)^2+\frac{a^2}{2}\cdot(\sqrt{-\Delta}\cdot\hat{u}(\vec{x},t))^2\right]:d^3x. \tag{14}$$

The result equals to

$$\hat{H}=\int a\cdot|\vec{p}|\cdot\hat{c}^+(\vec{p})\cdot\hat{c}(\vec{p})\cdot d^3p \tag{15}$$

and exactly coincides with Hamiltonian of massless scalar field (9) [3]

$$\hat{H}=\int:\left[\frac{1}{2}\cdot\left(\frac{\partial\hat{u}(\vec{x},t)}{\partial t}\right)^2+\frac{a^2}{2}\cdot(\nabla\hat{u}(\vec{x},t))^2\right]:d^3x \tag{16}$$

acting in Fock space.

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Feynmanons in the Korteweg-de Vries equation

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It is well known that Cauchy problem for the Korteweg-de Vries (KdV) equation:

$$\frac{\partial u}{\partial t} - 6\cdot u\cdot\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0, \quad t > 0, \quad -\infty < x < +\infty, \quad u(x,0) = u_0(x), \tag{1}$$

describes a wide range of physical phenomena [1]. In order to find exact solution of the KdV equation (1) one ought to solve the Gelfand-Levitan-Marchenko (GLM) equation [1]:

$$K(x,y;t) + B(x+y;t) + \int_x^{+\infty} B(y+z;t)\cdot K(x,z;t)\cdot dz = 0\,. \tag{2}$$

Let us now consider the situation when initial condition $u_0(x)$ in (1) is the potential hump. In this case stationary Schrödinger equation connected with this potential $u_0(x)$:

$$\frac{d^2\cdot}{dx^2} + (\text{ª} - u_0(x))\cdot\cdot\; = 0 \tag{3}$$

has no discrete spectrum. Therefore the kernel $B(x,t)$ of the GLM linear integral equation (1) can be expressed as [1]:

$$B(x,t) = \int_{-\infty}^{+\infty} b_0(p)\cdot\exp(i\cdot p\cdot x + 8\cdot i\cdot p^3\cdot t)\cdot\frac{dp}{2\cdot\text{®}}\,, \tag{4}$$

where $b_0(p)$ is reflection coefficient for equation (3).

It is easy to see that kernel (4) obeys to the following linearized KdV equation:

$$\frac{\partial B}{\partial t} + 8\cdot\frac{\partial^3 B}{\partial x^3} = 0\,. \tag{5}$$

Thus function (4) is equal to convolution of Fourier transform $B(x,0)$ of reflection coefficient $b_0(p)$ with Green's function of equation (5):

$$B(x,t) = \int_{-\infty}^{+\infty} G(x-x';t)\cdot B(x',0)\cdot dx'\,, \tag{6}$$

which can be expressed through the well-known Airy function:

$$G(x;t) = \frac{1}{2\cdot\sqrt[3]{3\cdot t}}\cdot Ai\left[\frac{x}{2\cdot\sqrt[3]{3\cdot t}}\right]. \tag{7}$$

On the other side equation (5) can be rewritten as nonstationary equation of Schrödinger-like type namely:

$$i\cdot\frac{\partial B}{\partial t} = \hat{H}B \tag{8}$$

with Hamiltonian $\hat{H} = -8\cdot\hat{p}^3$, where $\hat{p} = -i\cdot\partial/\partial x$ is operator of momentum.

It means that quite similarly to Green's function of Schrödinger equation for free particle Green's function (7) can be represented by the following Feynman integral:

$$\frac{1}{2\cdot\sqrt[3]{3\cdot t}}\cdot Ai\left[\frac{x-x'}{2\cdot\sqrt[3]{3\cdot t}}\right]=\int\limits_{Q(0)=x'}^{Q(t)=x}\exp\left[i\cdot\int\limits_0^t(P(\tau)\cdot\dot{Q}(\tau)+8\cdot P^3(\tau))\cdot d\tau\right]\cdot\prod_{\tau}\frac{dP(\tau)\cdot dQ(\tau)}{2\cdot\pi}. \quad (9)$$

Due to this expression we take the opportunity to introduce quantum quasiparticle related with equation (5) as object moving along trajectories in two-dimensional phase space (P,Q) and to call this quasiparticle by ‘feynmanon’. In report [2] it is shown that in these situations the large majority of these trajectories are fractal. Furthermore dynamics of momenta proves to obey to succession map $\overline{P}=f(P)$ which may possess by chaotic behaviour and may be closely related with fractional derivatives.

Moreover the kernel of the GLM equation has another Feynman integral because two-dimensional plane wave in formula (4) equals to [3]:

$$\frac{\exp(i\cdot\vec{k}\cdot\vec{x})}{-2\cdot\pi\cdot i}=\int\limits_{\vec{Q}(-\pi/2)=\vec{k}}^{\vec{Q}(0)=\vec{x}}\exp\left[-i\cdot\int\limits_{-\pi/2}^{0}\sum_{j=1}^{2}\left(P_j(\tau)\cdot\dot{Q}_j(\tau)-\frac{P_j^2(\tau)+Q_j^2(\tau)}{2}\right)\cdot d\tau\right]\cdot d«, \quad (10)$$

where $\vec{x}=(x,t)$, $\vec{k}=(p,8\cdot p^3)$ and

$$d«=\prod_{j=1}^{2}\prod_{\tau}\frac{dP_j(\tau)\cdot dQ_j(\tau)}{2\cdot\pi} \quad (11)$$

is Feynman’s pseudomeasure in four-dimensional phase space $(\vec{P},\vec{Q})$.

The exact solution of problem (1) can be expressed through the solution $K(x,y;t)$ of the GLM equation (2) as follows [1]:

$$u(x,t)=-2\cdot\frac{\partial K(x,x;t)}{\partial x}. \quad (12)$$

It means that feynmanons penetrate very deep into the KdV equation may be because of Schrödinger equation (3) in starting point of our analysis.

In conclusion it is necessary to underline that one can find from generalized uncertainty relations [4] that under some assumptions solution of equation (5) obeys to the following inequality (parameters x_0 and p_0 are positive):

$$\int\limits_{-x_0}^{+x_0}|B(x,t)|^2\,dx\le\cos^2\left[\arccos\sqrt{\lambda_0(p_0\cdot x_0)}-\arccos\sqrt{\int\limits_{-p_0}^{+p_0}|b_0(p)|^2\,\frac{dp}{2\pi}}\right], \quad (13)$$

where $\lambda_0(c)$ is maximal eigenvalue of integral equation:

$$\int\limits_{-1}^{+1}\frac{\sin[c\cdot(x-x')]}{\pi\cdot(x-x')}\cdot f(x')\cdot dx'=\lambda(c)\cdot f(x) \quad (14)$$

for prolate spheroidal wave functions [4].

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Global well-posedness and attractors for the hyperbolic Cahn-Hilliard-Oono equation in the whole space

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The talk is devoted to the so called hyperbolic relaxation of Cahn-Hilliard-Oono equation in $\mathbf{R}^3$ with sub-quintic non-linearity. Based on Strichartz estimates for Schrodinger equation the global well-posedness for the original problem is proven that drastically improves admissible growth of the nonlinearity known before. Furthermore, existence of the compact global attractor for the corresponding semi-group, its smoothness and finite fractal dimensionality are established. If time permits similar results related to the damped wave equation will be discussed. The work is joint with Prof. Sergey Zelik.

A gradient flow approach to a fractional porous medium equation

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In this seminar I will describe how the following fractional porous medium equation, recently introduced and studied by Caffarelli & Vázquez,

$$\begin{cases} \partial_t u - \operatorname{div}(u\nabla v) = 0 \quad \text{in } \mathbb{R}^d \times (0,+\infty), \\ (\alpha I - \Delta)^s v = u \quad \text{in } \mathbb{R}^d \times (0,+\infty), \quad s \in (0,1), \quad \text{and } \alpha \geq 0, \end{cases}$$

can be interpreted as a gradient flow in the space of probability measures endowed with the Wasserstein distance.

This is a joint project with S. Lisini (Pavia) and E. Mainini (Genova).