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# **“Explanatory Potential of Mathematics in Quantum Physics”**

International conference “New mathematical methods in today’s physics:  
logical, epistemological and computational aspects”

Institute of Philosophy, Russian Academy of Sciences

21-22 September 2015

# Introduction

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The mathematical equations have an amazing property. Real systems follow equations with unexplained persistence. It is the part of the overall problem of “unreasonable effectiveness of mathematics” in the description of reality.

In classical physics, most of the equations of motion corresponds to one of the physical theories. These theories give us some models and interpretations connecting the equations with reality.

In quantum physics, the situation is more complicated.

I assume that in the interpretation of QM, we cannot ignore the explanatory potential of mathematical formalisms and their mathematical concepts.

Quantum equations can be seen not only as tools for calculation. We can find them a way to explain reality. At least, we have to try. Exactly how we find the explanation of reality in the equations of Newtonian mechanics, thermodynamics or electrodynamics.

# Introduction

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In physics, there are widely-spread notions like these: ***possible, virtual, or imaginary states, events or histories***. These notions are applied in the variational principles, interpretations of quantum mechanics, Feynman path integral, and quantum cosmology.

On the one hand, it is accepted that all such notions are merely formal mathematical tools for calculation. On the other hand, there is the question: ***Why are the models and formalisms with such notions more successful than others?***

The natural desire is to use the models of QM and its equations for the explanation of their physical meaning. However, there are two obstacles.

**First**, these ***formalisms use too abstract mathematical concepts***. These concepts are difficult explained in frameworks of the older theories. It concerns the wave function, quantum action, probability amplitude, quantum operators, Hilbert space, quantum superposition and entanglement.

# Three formalisms of QM

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The second obstacle is that **QM includes several equivalent models, and each of them has their own formalism and own ontology.** Let us take three mathematically equivalent formalisms of QM based on the different models.

**Heisenberg** agreed with the Copenhagen interpretation, but he added a metaphysical idea of Aristotle about a transition from potential state to actual.

**Schrödinger** was convinced that the wave function is associated with the actual wave that carries an electrical charge and unambiguously describes an evolution of a quantum system.

**Feynman path integral** is a geometric model of a summation of rotating arrows. The arrows symbolize probability amplitude of virtual or possible paths of quantum particles. The probability of quantum events could be found by summing all contributions of all possible probability amplitudes, and then squaring this sum. Feynman interpreted his model using an analogy with the classical principle of least action. For this, he represented a particle that simultaneously moves along all alternative virtual paths.

# What is interpretation?

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All three formalisms are in good agreement with the experiments. **But the views of reality of their creators was very different.** So they were insufficient to explain the nature of quantum probability and the meaning of the wave function.

**The questions about the mathematical concepts.** What are the physical and philosophical status of the virtual histories? How is the classical action related to the quantum action? What are the physical and philosophical content of complex variables? How does the transition occur from the quantum probability amplitude to the classical probability?

**The main goals of the interpretation:**

- (a) to find out which natural phenomena are hidden behind the equations;
- (b) to describe all parts of the theory in terms of existence and reality;
- (c) to explain the relationships of these parts with an experiment.

Any complete theory is a set of the **theoretical part** including the ontology of theory plus its model, **mathematical formalism, experiments**, and the **interpretation of all these parts.**

# Possibilities in QM

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In many interpretations of QM, **possible states, events and histories of quantum particles are directly compared with possibilities**. In many cases, the quantum world or worlds are presented as the sets of *the possible events* or *possible histories* of the quantum fields.

Nowadays, the quasi-modal approach, which divides the reality into possible and actual realms, is quite popular in QM (*Fock, 1957; Everett, 1957; Popper, 1990; Wallace, 2003; Bohm, 1980; Van Fraassen, 1991; Gell-Mann & Hartle, 2012; Dieks, 2007; Lombardi & Castagnino, 2008; Suárez, 2011*).

However, the authors share very different attitude to the reality of these possible events and histories.

Recently, there has been a growing interest in **a realistic interpretation of the Feynman paths and alternative quantum histories** in addressing the problem of quantum reality (*Belnap, 2007; Sharlow, 2007; Wharton et al., 2011; Gell-Mann & Hartle 2012*).

# What about philosophy?

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It is known that modern philosophers also investigate some concepts that can be connected with the possible histories. For instance: *possibilities, pure possibilia, possible scenarios, and possible worlds*.

However, in spite of several interesting attempts to combine some of these notions in both quantum physics and metaphysics (*Redhead, 1987; Saunders, 1998; Wallace, 2003; Wilson, 2006; Dieks, 2010*), scholars have not yet adequately addressed two central questions:

- Are the possible histories in physics and possibilities in metaphysics real?
- What is common and different between the reality of the *possible histories* in classical physics, quantum physics, and metaphysics?

Before to analyze the reality of the quantum possible events and histories, let us consider a realistic and anti-realistic approaches to the mathematical concepts.

# Anti-realism of the mathematical concepts

There are several ways to explain why the predictions of the equations are consistent with the observed movement. I call only three.

## **Anti-realism.**

In the phenomena, there are no order and logic. People observe the phenomena and create some mathematical models to describe the observations. They use the human logic and human mathematical language. These logic and language are common. Thus, it is not surprisingly that the results of the different people are the same. This is considered as the reason of the effectiveness of mathematics. We don't know the true reality.

There are only our perceptions, approximations and logic constructions. **Any theory is only a temporary model that is suitable to describe a certain kind of the phenomena.**

Mathematical concepts exist only in our minds as the tools of cognition. The equations describe only our experience (**Instrumentalism**).



# Realism of the mathematical concepts

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The mathematical models are ontological and reflect the real relationships between essences of the phenomena. The proponents of **scientific realism** believe that the good theory and its equations say about things and events that exist and occur in fact, regardless of our mind. However, in realism, there are two possible views of reality of the mathematical concepts.

- a) Mathematical concepts exist only in our minds as the soft cognition (**Instrumentalism**).
- b) Mathematical concepts ontologically exist as independent entities (**Platonism**).

The position of the **anti-realist** is contradicted to our common sense. However, **from a practical point of view, it is more convenient**. Perhaps, that is why anti-realism is so popular.

The **realist** faces more difficulties. He has to make a choice between the equations of different models and decide which of these describe the reality.

# Between Realism and Anti-realism

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We and our consciousness are involved in the creation of reality. We are not separate from the world.

Some mathematical concepts ontologically exist, but they depend on our bodies and minds. It is because our bodies and minds are the tools of cognition and change of reality.

However, not every model describes the essence behind the phenomena. Some mathematical formalisms describe only phenomenological laws of specific areas of nature. Others reflect the real relationships between the objects and phenomena.

Mathematics in nature is always richer mathematics in our minds.

I call this view – **semi-realism**.

# Anti-realism vs. Realism in QM

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Let us introduce the definition: **the possible quantum event or history** is the event or history that is consistent with the laws of QM.

**Quantum possible world** is the set of the possible events or histories of the quantum fields.

**Anti-realism:** the possible quantum events exist only in models and formulas, as tools of the theoretical study of our experience. They do not have an independent life. Quantum world does not exist. It is only possible scenarios that are potentially contained in the classical objects.

**Realism and Semi-realism:** the quantum possible events or histories exist independently of our mind. They are the manifestation of objective propensities or potentialities - dispositions (**essential dispositionalism**). The quantum possible world is a set of possible events or histories that are real as well as our world for us (**modal realism**). Quantum possible world ontologically looks like a set of potential events or histories that have not been realized and yet do not exist in our actual world (**possibilism**).

# Four groups of the interpretations of QM

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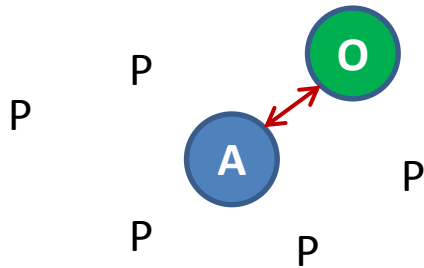
Let us consider **how the possible events and possible histories can be used for the interpretation of QM**. These can be united in four groups. The principle of the division is based on how the possible states and possible histories are related with the actual ones. **In every group, we can find the points of view, which follow anti-realism, realism or semi-realism.**

- **In the first group**, one of the possible quantum events or histories becomes actual as a result of their observation.
- **In the second group**, one of the possible quantum events or histories is merely detected by the measurement or by the interaction. Observation means are relative in relation to the objects.
- **In the third group**, each possible quantum event or history is realized as actual.
- **In the fourth group**, a certain set of possible quantum events or histories is realized at once. We observe the actual result of their combination.

# The first group of interpretations of QM

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## 1st group



One of the possible quantum events or histories becomes actual as a result of their observation.

# The first group of interpretations of QM

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**Bohr's version of Copenhagen interpretation.** It is meaningless to talk about the reality of the possible states of quantum particles before their measurement because these exist only in formulas. The only actual world is created by the measurement, and the “collapse of a wave function” does not describe a change the reality, but a change of our knowledge of the reality (**anti-realism**).

**Heisenberg** believed that mathematical laws of quantum theory can be considered in Aristotelian notions of “*dynamis*” or “*potency*”, and that a notion of “possibility” occupies an intermediate position between objective material reality and subjective reality (**semi-realism**).

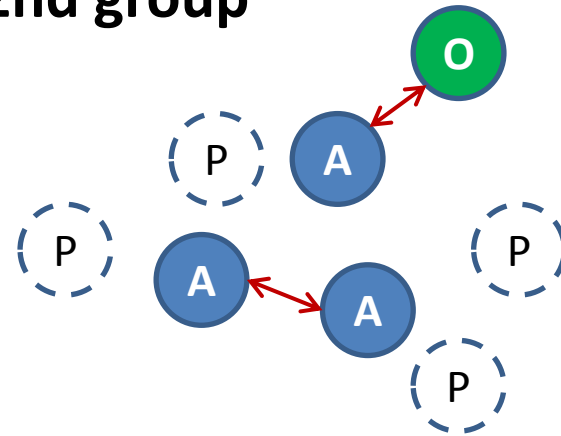
**Von Neumann and Wigner** believed that the consciousness of the observer, which is connected with devices, creates a reality, destroying the superposition of possible states (**semi-realism**).

**Wheeler** considered the being of the whole universe as a result of the participation of an observer in the process of self-realization of the universe, through the exchange of information (**semi-realism**)

# The second group of interpretations of QM

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2nd group



One of the possible quantum events or histories is merely detected by the measurement or by the interaction.

The observation means are relative in relation to the objects.

# The second group of interpretations of QM

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In contrast to Heisenberg, Fock believed that the state, which described by the wave function is objective. It is characteristic of the potential possibilities of one or another result of the interaction between the object and a device (**semi-realism**).

Oge Bohr (son). Quantum reality is described by operators rather than numbers. Quantum reality contains the whole spectrum at once real worlds (**realism** and **platonism**).

Van Fraassen proposed that a quantum system has two kinds of states: *dynamical state* and *value state*. The dynamical state determines the system's possible physical properties and their probabilities. The value state represents actual physical properties. The measurement as well as any physical interaction randomly detects (**but does not create, as Heisenberg supposed**) one of the possible value states and makes it actual. The quantum formalism does not say what actually happens in the physical world, but it only gives us a list of the possibilities and their probabilities (**semi-realism**).



# The second group of interpretations of QM

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**Modal interpretations (Bene and Dieks).** Modalities are mere convenient tools for the description of the actual world and do not have their existence (**anti-realism**).

**Modal-Hamiltonian interpretation (Lombardi and Castagnino)** introduce ontology with irreducible to each other realm of possibility and realm of actuality. Quantum systems are within the realm of possibility that is not less real than the realm of actuality. The propensities follow a deterministic evolution independently of which possible facts become actual. The propensities produce effects on the actual reality even if they never become actual (**realism** and **platonism**).

**Consistent Histories (Gell-Mann and Hartle)** select from all alternative quantum histories (Feynman paths) a set of coarse-grained coherent histories. Due to decoherence or “entanglement with the environment” only part of them interfere with each other. The reality before decoherence does not depend on the measurement. The possible histories that turn to actuality under certain conditions (**semi-realism**).

# The second group of interpretations of QM

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**Existential interpretation (Zurek)** based on a mechanism of decoherence with the environment. It combines two opposite point of view:

- (a) the reality is only our knowledge, as in the Copenhagen interpretation
- (b) the reality is an ontological entity.

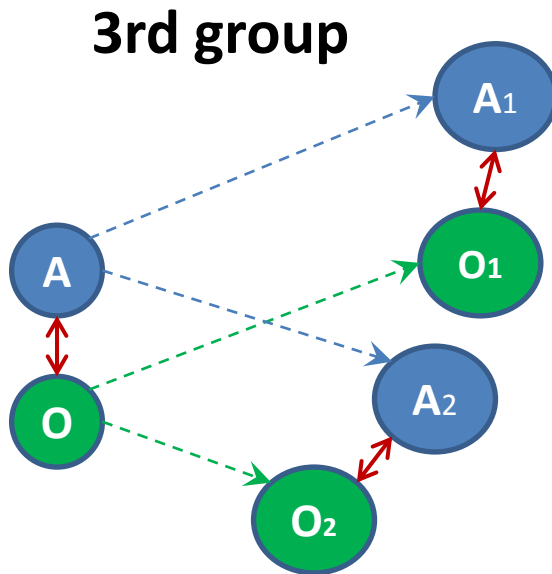
The ontological features of the actual states are selected only when the superposition principle is “turned off” by environment induced decoherence.

The objective existence of the selected states is acquired through the epistemological information exchange with the environment. This exchange of information exists objectively; it is the cause of any changes and interactions. It is supposed that information is not only human knowledge but the primary entity (**semi-realism**).

# The third group of interpretations of QM

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Each possible quantum event or history is realized as actual.



# The third group of interpretations of QM

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**Many-Worlds interpretation of QM.** Any measurement of quantum particles divides them into many copies. Each of copies actually exists in a parallel world or a projection of a multiverse. Multiverse is composed of a quantum superposition of all its own possible branches or quantum worlds. Any copy evolves according to the Schrödinger's equation, and a wave function is an ontological entity.

At first glance, this interpretation resembles the conception of modal realism in metaphysics of possible worlds where all possible worlds exist and are relatively actual. However, the similarity is deceptive (Lewis, 2004).

In modal realism, possible worlds or universes develop independently of each other even under different laws. In the Many-Worlds interpretation, each possible world is mere one of all possible alternative histories or branches of the evolution of the single multiverse.

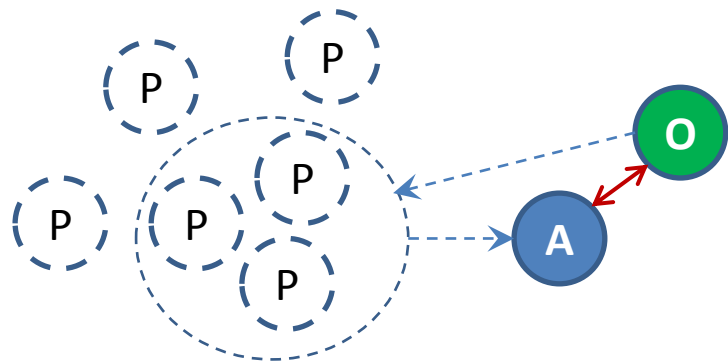
Everett denied any direct analogy with the transition the possibility to actuality that adopted in possibilism (**realism**).

# The fourth group of interpretations of QM

A certain set of possible quantum events or histories is realized at once.

We observe the actual result of their combination.

4th group



# The fourth group of interpretations of QM

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**Schrödinger** explained the actual trajectory by a set or a field of all possible trajectories. In an infinite number of possible trajectories none of these has the advantage to be implemented in a particular case, all these are equally real.

Instead of the implementation of only one possible entangled state (**Heisenberg**), all ones are summed up. **It occurs due to resonance or interference of the waves (semi-realism).**

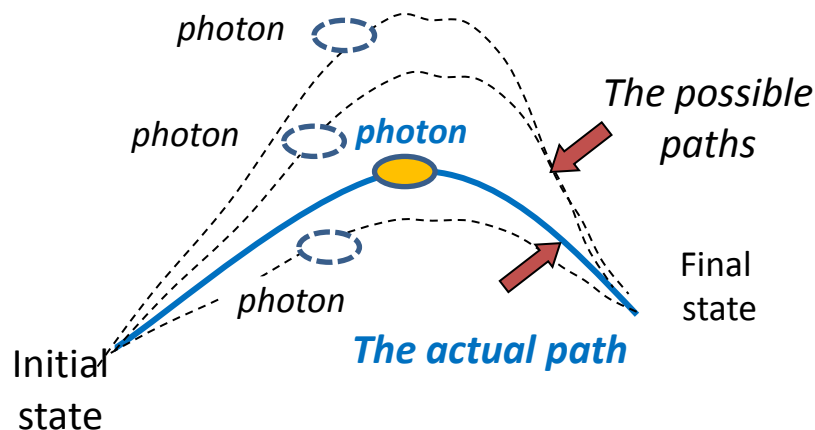
**De Broglie-Bohm.** Non-local in space-time field of quantum potentials objectively exists independently of consciousness as a set of features. This non-local field depends on positions of all particles that at once influence the actual trajectory of the particle.

According to Bohm's theory of holomovement, any measurement or interaction extracts the objects from an entangled state of "*undivided wholeness*" and "*implicate order*" (**semi-realism**).

# The fourth group of interpretations of QM

**Feynman path integral.** The photons do follow along all possible paths, and the summation of their probability amplitudes is not empty play in mathematics. It seems that a particle “feels” all the neighboring trajectories and selects the one along which the action is minimal.

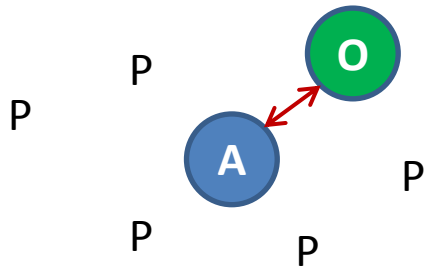
In our classical world, these possible histories are mutually-exclusive, although, at the quantum level these possible histories coexist in quantum superposition (**semi-realism**). This view of the reality is supported by an **analogy with the classical principle of least action**.



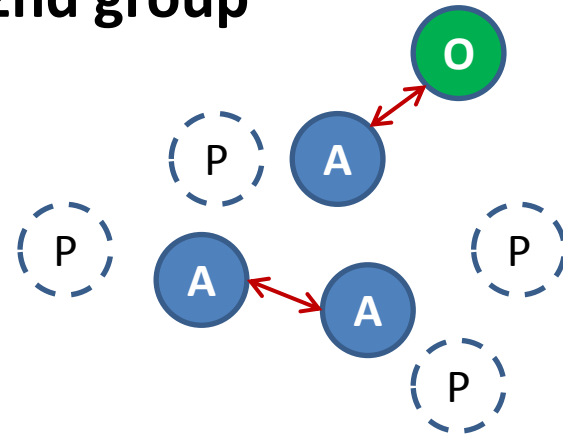
**The probability of the actual path = maximum**

# Four groups of interpretations of QM

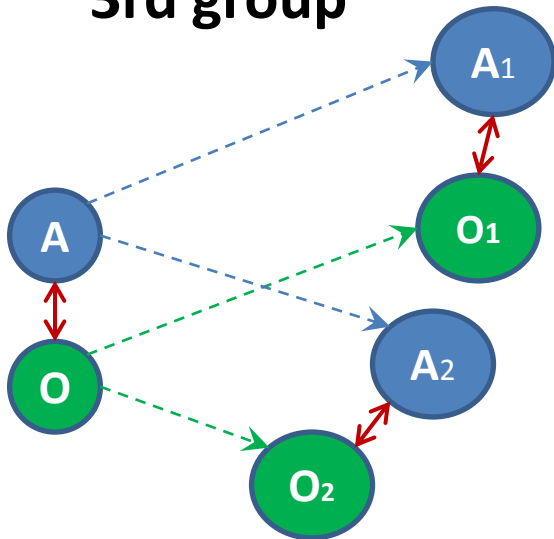
1st group



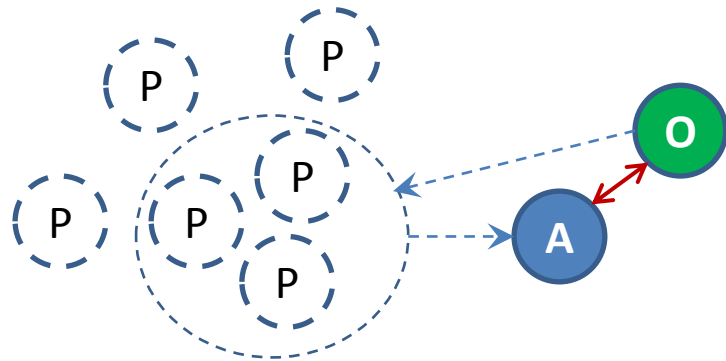
2nd group



3rd group



4th group





# Conclusion

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In every group, we can find the points of view that follow **anti-realism, realism or semi-realism.**

Most of the interpretation of QM can be attributed to the position of **semi-realism**. It means that **most of the physicists consider the possible quantum states or histories as having a certain grade of reality.**

It seems they believe that some mathematical concepts **reflect the real relationships between the concepts and phenomena.**

# Conclusion

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The objections in order to use the mathematical formalisms for the explanation of quantum phenomena.

**Anti-realist:** the mathematical concepts exist only in our minds as the tools of cognition. However, it is not clear:  
What is the source of reality of our mind, which conceives something?  
Why do we conceive the same classical world?  
Whether the laws of nature exist or not exist?

**Scientific realist** is not against the transfer of the mathematical concepts to reality. But he agrees only with interpretation, which has a clear physical meaning. He argues that the variational principles and path integral do not have any physical meaning. These are merely convenient metaphors. However, **the presence or absence of the physical meaning are our interpretations too.**

# Conclusion

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In the interpretation of QM, we cannot ignore the explanatory potential of mathematical formalisms and their concepts.

Quantum equations can be seen not only as tools for calculation. We can find them a way to explain reality. At least, we have to try.

If we assume that mathematics is not limited by our brain, but in some degree reflects the real processes, **we should use the mathematical concepts and operations to develop our ontological views of reality.**

Maybe, the ordered ontological ideas will help mathematicians to explain the meaning of the familiar mathematical concepts.



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